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Prediction of Delta Wing Leading-Edge Vortex Circulation and Lift-Curve Slope

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Nomenclature

b = wing span
C_T = leading-edge thrust coefficient
c = chord
c_s = sectional leading-edge suction coefficient
c_t = sectional leading-edge thrust coefficient
k_p = potential constant
q = dynamic pressure
S = wing area
t = sectional leading-edge thrust
U_∞ = freestream velocity
w = downwash velocity
x = chordwise direction
y = spanwise direction
α = angle of attack
Γ = vortex circulation
γ = vorticity per unit length
ε = wing apex half-angle
Λ = leading-edge sweep angle
ρ = density

Subscripts

L.E. = leading edge
v = vortex

Introduction

THE leading-edge suction analogy of Polhamus[1] provides an accurate means by which the aerodynamics of slender sharp wings may be estimated. Slender swept wings typically have flowfields dominated by conical leading-edge vortices. The basic tenet of the suction analogy is that the leading-edge suction force through enforced leading-edge flow separation, in combination with sweep, is effectively rotated through 90 deg to the plane of the normal force, and manifests as the force required to effectively maintain equilibrium of the vortex above the wing. The analogy does not, however, yield information on the leading-edge suction distribution or vortex characteristics.

Use of a panel method allows determination of the leading-edge thrust or suction distribution by using the computed attached flow sectional lift and vortex drag at each spanwise station. The leading-edge suction distribution may also be determined analytically, using an expression derived by Purvis[2] for arbitrary planforms. To calculate the strength of the leading-edge vortex, a panel method may be employed, with various methodologies being used to estimate the vortex strength,[3] e.g., calculating the velocity above and below the leading-edge wake. Euler and Navier–Stokes solvers can also be used to determine the vortex properties, but are computationally expensive and sensitive to the grid employed.[4] None of these computational methods, however, show explicitly the functional relationship of vortex strength to parameters such as wing leading-edge sweep, α, and chordwise location.

Helmholtz' vortex theorems ensure that the rate of change of the spanwise load distribution relates to the rate at which vorticity is shed from a wing's trailing edge ($\gamma = -\mathrm{d}\Gamma/\mathrm{d}y$). Thus, it may be analogous to assume that the rate of change of the leading-edge thrust distribution relates to the rate at which vorticity is shed from the leading edge, and consequently, into the leading-edge vortex. Using this analogy, combined with the aforementioned expression of Purvis[2] for the leading-edge suction distribution, allows the derivation of an expression to estimate the leading-edge vortex chordwise circulation distribution. In this Note, an expression is derived to estimate the strength of the leading-edge vortex of delta wings. The vorticity shed from the wing leading edge is related to the rate of change of the wing leading-edge thrust, yielding an expression for γ, which can be integrated to yield vortex circulation. The form of this expression allows derivation of two equations for the potential constant or attached flow lift–curve slope of delta wings.

Hemsch and Luckring[5] have derived an expression using a Sychev similarity parameter to estimate the strength of the leading-edge vortex at the wing trailing edge. Using this expression it is possible to estimate the strength of the vortex along the wing. This does, however, require the assumption of conical flow (i.e., $\Gamma \propto x$), and two empirical constants.

Discussion of Method

Using the Kutta–Joukowski theorem to calculate the leading-edge thrust associated with the freestream parallel to the chord gives

$$\mathrm{d}t = \rho(U_\infty \sin\alpha - w)\frac{\mathrm{d}\Gamma}{\mathrm{d}y}\,\mathrm{d}y \tag{1}$$

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where w is the downwash associated with the trailing vortex system. Thus,

$$\frac{d\Gamma}{dy} = \gamma = \frac{dt}{dy}\frac{1}{\rho(U_\infty \sin\alpha - w)} \tag{2}$$

$U_\infty \sin\alpha - w$ should effectively be the upwash distribution at the wing's leading edge. However, for the purposes of the present study, it is not feasible to estimate the local variation of downwash at the leading edge. The upwash distribution may be estimated by adapting Purvis'[2] approach to the approximation of $U_\infty \sin\alpha - w$ by allowing it to be proportional to the approximation from Ref. 2, i.e.,

$$U_\infty \sin\alpha - w = K(U_\infty C_T/k_p \sin\alpha) \tag{3}$$

where K is a constant of proportionality.

Noting that the leading-edge thrust is related to the leading-edge suction by $cc_t = cc_s \cos(\Lambda)$, gives

$$\gamma = \frac{k_p \sin\alpha \cos\Lambda}{\rho K U_\infty C_T}\frac{\rho U_\infty^2}{2}\frac{dcc_s}{dy} \tag{4}$$

The leading-edge suction may be estimated as[2]

$$cc_s = \frac{SE_0 C_T}{\pi \cos\Lambda}\left[\frac{2y}{b}\sqrt{1-\left(\frac{2y}{b}\right)^2} + \arcsin\frac{2y}{b}\right] \tag{5}$$

Substituting Eq. (5) into Eq. (4) yields

$$\gamma = \frac{k_p \sin\alpha}{K}\frac{SE_0}{\pi}\frac{U_\infty}{2}\frac{d}{dy}\left[\frac{2y}{b}\sqrt{1-\left(\frac{2y}{b}\right)^2} + \arcsin\frac{2y}{b}\right] \tag{6}$$

which upon differentiating becomes

$$\gamma = \frac{U_\infty S k_p E_0 \sin\alpha}{2\pi K}\left[\frac{2}{b}\sqrt{1-\left(\frac{2y}{b}\right)^2} - \frac{8y^2}{b^3\sqrt{1-(2y/b)^2}} + \frac{2}{b\sqrt{1-(2y/b)^2}}\right] \tag{7}$$

Equation (7) shows that under the assumptions of this analysis, γ is proportional to $\sin(\alpha)$, or the velocity component normal to the wing as well as the potential lift–curve slope k_p. The wing leading-edge sweep does not enter directly into the expression but is effectively included through k_p.

To calculate the strength of the circulation at the leading edge at a specific y, Eq. (7) may be integrated using

$$\Gamma_{\text{L.E.}}(y) = \int_0^y \gamma\, dy \tag{8}$$

yielding, after substituting for S, and $E_0 = 1.106\pi/b$

$$\Gamma_{\text{L.E.}}(y) = \frac{1.106 c U_\infty k_p \sin\alpha}{4K}\left[\frac{2y}{b}\sqrt{1-\left(\frac{2y}{b}\right)^2} + \arcsin\frac{2y}{b}\right] \tag{9}$$

The constant K will be evaluated later using experimental data. For the present method to have more general application, i.e., computational prediction of the cc_t distribution, it is necessary to clarify some of its limitations. Panel method computation of cc_t will show this value tending to zero as the wingtip is approached [this characteristic is not present in Eq. (5)]. Thus, the thrust distribution increases from zero at the root, to a maximum near the wingtip, and then to zero. This

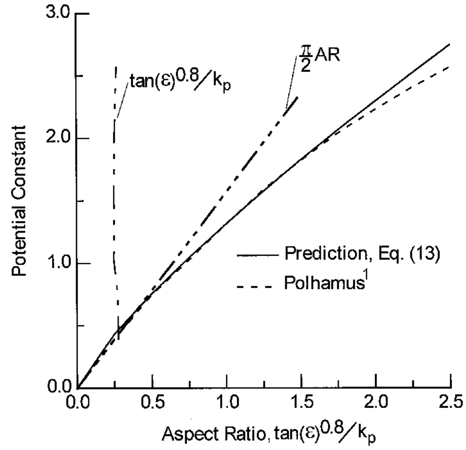

Fig. 1 Correlation of the potential lift-curve slope and wing apex angle for simple delta wings, allowing prediction of the potential constant as a function of AR.

would suggest that the strength of the vortex would increase due to vorticity from the leading edge and then start to decrease as the slope of dcc_t/dy, and hence, the vorticity shed changes sign. Euler[6] computations of the circulation at the leading edge and vortex core of a 70-deg delta wing show an essentially linear (or conical) increase in circulation, followed by a reduction in circulation as the wingtip is approached. Thus, although the trend of reducing leading-edge Γ is correct as the tip is approached, the present method would have $\Gamma_{\text{L.E.}}$ tend to zero (with cc_t) at $y = b/2$. This is obviously incorrect and would suggest that the vortex does not persist downstream. Note that this problem is not present in Eq. (9) due to the form of Eq. (5). A more general formulation for the vortex strength that encompasses the correct physics is given by Eq. (10):

$$\Gamma_v(y) = \int_0^y \frac{\Gamma_{\text{L.E.}}}{y}\, dy \tag{10}$$

This expression may be interpreted as summing the sectional leading-edge circulation to yield the total vortex circulation. Equation (10) tends to a constant $\Gamma_v(y)$ as the leading-edge thrust, and hence, circulation tends to 0.

Equation (9) shows that for a given spanwise location the vortex circulation is essentially proportional to the potential constant multiplied by a function of α. Using Hemsch's[5] expression for vortex circulation at the wing trailing edge, combined with Vissers'[7] experimental constant, allows determination of a simple estimate for the attached flow lift–curve slope of delta wings. Hemsch's expression may be written as

$$\Gamma/U_\infty c = 4.63 \tan^{0.8}\varepsilon \tan^{1.2}\alpha \cos\alpha \tag{11}$$

Noting that the present analysis shows that $(\Gamma/U_\infty c)[1/k_p f(\alpha)]$ should be a constant for a given spanwise location, and for consistency using the formulation from Ref. 5 for the dependence of Γ on α, i.e., $f(\alpha) = \tan^{1.2}\alpha \cos\alpha$, gives

$$\frac{4.63 \tan^{0.8}\varepsilon \tan^{1.2}\alpha \cos\alpha}{k_p \tan^{1.2}\alpha \cos\alpha} = \text{const}$$

or

$$\tan^{0.8}\varepsilon / k_p = \text{const} \tag{12}$$

Using Polhamus'[1] results for k_p of delta wings to see if Eq. (12) does indeed correlate leading-edge sweep and k_p (Fig. 1), shows that the data essentially collapse to a constant value of 0.25. This yields two simple expressions for the lift–curve slope of simple delta wings:

$$k_p = 4 \tan^{0.8}\varepsilon = \text{AR}/\tan^{0.2}\varepsilon \tag{13}$$

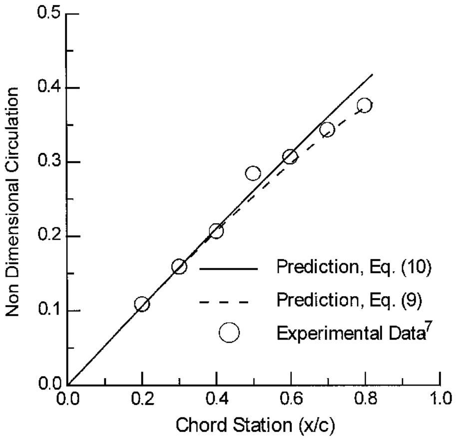

Fig. 2 Comparison of theory and experiment for chordwise variation of circulation, Λ = 75 deg, α = 20 deg.

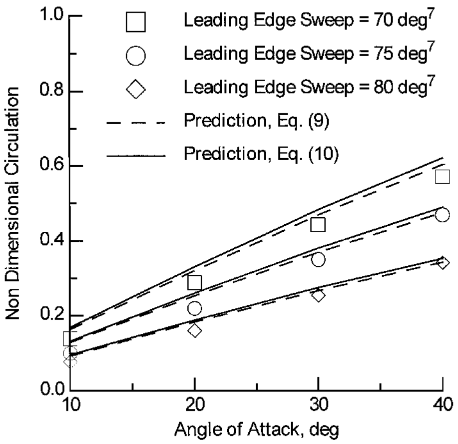

Fig. 3 Comparison of theory and experiment for variation of circulation with sweep and incidence, x/c = 0.5.

The similarity of the form of Eq. (13) to that for the AR of a delta (AR = 4 tan ε) is interesting. Figure 1 also shows comparisons of k_p calculated using Eq. (13) and data from Ref. 1; it may be seen that Eq. (13) demonstrates excellent accuracy for AR < 2, as would be expected. Also included in the figure is the result for the lift–curve slope of slender deltas as per slender wing theory, i.e., $k_p = (\pi/2)$AR.

To estimate the constant K for delta wings, comparisons of Eqs. (9) and (10) with experimental results of Visser and Nelson[7] are shown in Figs. 2 and 3. The circulation values presented are nondimensionalized by the freestream and wing root chord. Setting K = 0.5 yields the correlations shown. Figure 2 shows that the present method displays very good correlation with the experimentally determined chordwise vortex circulation distribution. Notice that over the forward part of the wing, Eqs. (9) and (10) are in good agreement. As the trailing edge is approached, Eq. (10) tends to predict higher circulation values, as would be expected noting its formulation. Although not directly modeling the physics, Eqs. (9) and (10) do predict the correct trends,[6] i.e., over the forward part of the wing the leading-edge circulation and vortex circulation are similar; however, as the trailing edge is approached the vortex circulation increases beyond that of the leading-edge circulation. Figure 3 shows that the method closely predicts the trends of sweep and α on $\Gamma/U_\infty c$. The experimental circulation shown is the maximum measured at the x/c = 0.5 location. The predicted results in Fig. 3 were calculated by evaluating Eq. (9) at $b/4$ (i.e., x/c = 0.5), and integrating Eq. (10) spanwise from the wing root to $b/4$.

Concluding Remarks

A method is presented that predicts the leading-edge vorticity and vortex circulation for delta wings. The method relates the strength of the vorticity shed to the rate of change of leading-edge thrust. Circulation was calculated by both integrating the vorticity distribution as well as integrating the sectional circulation distribution. The study suggests that the leading-edge vortex strength is proportional to the normal component of the freestream and the wing lift–curve slope, with sweep effects effectively being included in the lift–curve slope. The dependence of vortex strength on the potential constant and angle of attack also allows determination of two simple expressions for the attached flow lift–curve slope of delta wings.

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Impact of Initial Conditions on Vortex Breakdown on Pitching Delta Wings

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Introduction

WHEN comparing the experimental results[1] at $\bar{\omega}$ = 0.10 for a 52-deg delta wing (Fig. 1) with those for $\bar{\omega}$ = 0.107 (Fig. 2) for a 65-deg delta wing,[2] the following question arises. Why can the upstroke characteristics in Fig. 1 hardly get downstream of the static characteristics, especially considering that even during the initial portion of the downstroke the vortex breakdown in Fig. 2 prefers to stay on the downstream side of the static characteristics? These vortex breakdown characteristics[2] were analyzed in Ref. 3 by first considering the characteristics for purely rampwise changes of the angle of

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